Lecture Notes January 6, 2015

Reading assignment for this lecture: Syllabus, section I.

Economic General Equilibrium

Partial and General Economic Equilibrium
  PARTIAL EQUILIBRIUM

  $S_k(p^o_k) = D_k(p^o_k),$ with $p^o_k > 0$ (or possibly, $p^o_k = 0$), or

  $p^o_k = 0$ if $S_k(p^o_k) > D_k(p^o_k).$
  GENERAL EQUILIBRIUM   For all i = 1,...,N,
  $D_i(p^o_1, p^o_2,...,p^o_N) = S_i(p^o_1,..,p^o_N),$ $p^o_i \geq 0,$ and
  $p^o_i = 0$ for goods i such that $D_i(p^o_1,...,p^o_N) < S_i(p^o_1,...,p^o_N).$

Partial equilibrium assumes "other things being equal," that the variations
considered are all local and interactions are negligible. What's wrong with partial
equilibrium?
  > Suppose there's no consistent choice of $(p^o_1,..,p^o_N).$ Then there would be
(apparent) partial equilibrium --- viewing each market separately --- but no way to
sustain it, because of cross-market interaction.
  > Competitive equilibrium is supposed to make efficient use of resources
through optimizing firm and consumer choice where prices indicate scarcity. But
to verify this notion in partial equilibrium we must assume (<u>without proof</u>) that
prices in other markets reflect underlying scarcity.   If not, then apparently efficient
equilibrium allocation may be wasteful. A valid notion of equilibrium and
efficiency needs to take cross-market interaction into account.

Three big ideas
  Equilibrium: $S(p) = D(p)$, where S, D, and p are all N-dimensional vectors
  Decentralization
  Efficiency

The Edgeworth Box and the Robinson Crusoe model are two oversimplied
examples in which most principal observations of the general equilibrium theory
can be illustrated.

The Edgeworth Box
2 person, 2 good, pure exchange economy

Fixed positive quantities of X and Y, and two households, 1 and 2.

Household 1 is endowed with $\bar{X}^1$ of good X and $\bar{Y}^1$ of good Y, utility function
$U^1(X^1, Y^1)$. Household 2 is endowed with $\bar{X}^2$ of good X and $\bar{Y}^2$ of good Y, utility
function $U^2(X^2, Y^2)$

$$X^1 + X^2 = \bar{X}^1 + \bar{X}^2 \equiv \bar{X},$$
$$Y^1 + Y^2 = \bar{Y}^1 + \bar{Y}^2 \equiv \bar{Y}.$$

Each point in the Edgeworth box represents an attainable allocation of X^1 and X^2,
Y^1 and Y^2, consistent with the total resource endowment $(\bar{X}, \bar{Y})$.
1's origin is at the southwest corner; 1's consumption increases as the allocation
point moves in a northeast direction; 2's increases as the allocation point moves in
a southwest direction. Superimpose indifference curves on the Edgeworth Box.

Competitive Equilibrium in the Edgeworth Box
       (p^o_x , p^o_y) so that (X^{o1}, Y^{o1}) maximizes $U^1(X^1, Y^1)$ subject to
$(p^o_x , p^o_y)\cdot(X^1, Y^1) \leq (p^o_x , p^o_y)\cdot(\bar{X}^1, \bar{Y}^1)$ and
       (X^{o2}, Y^{o2}) maximizes $U^2(X^2, Y^2)$ subject to
$(p^o_x , p^o_y)\cdot(X^1, Y^1) \leq (p^o_x , p^o_y)\cdot(\bar{X}^2, \bar{Y}^2)$, and
       $(X^{o1}, Y^{o1}) + (X^{o2}, Y^{o2}) = (\bar{X}^1, \bar{Y}^1) + (\bar{X}^2, \bar{Y}^2)$
or $(X^{o1}, Y^{o1}) + (X^{o2}, Y^{o2}) \leq (\bar{X}^1, \bar{Y}^1) + (\bar{X}^2, \bar{Y}^2)$, where the inequality holds
co-ordinatewise and any good for which there is a strict inequality has a price of 0.

Pareto efficiency in the Edgeworth Box
       An allocation is Pareto efficient if all of the opportunities for mutually
desirable reallocation have been fully used. The allocation is Pareto efficient if
there is no available reallocation that can improve the utility level of one household
while not reducing the utility of any household.

Assuming convexity and monotonicity of preferences (quasi-concavity and
non-satiation of utility functions) and an interior solution, Pareto efficiency is
achieved at an allocation where there is tangency of 1 and 2's indifference curves.

Pareto efficient allocation: (X^{o1}, Y^{o1}), (X^{o2}, Y^{o2}) maximizes $U^1(X^1, Y^1)$ subject to $U^2(X^2, Y^2) \geq U^{o2}$ (typically --- assuming non-satiation --- equality will hold and $U^{o2} = U^2(X^{o2}, Y^{o2})$) and subject to the resource constraints

$$X^1 + X^2 = \bar{X}^1 + \bar{X}^2 \equiv \bar{X} \ ; \quad Y^1 + Y^2 = \bar{Y}^1 + \bar{Y}^2 \equiv \bar{Y} \ .$$

Equivalently, $X^2 = \bar{X} - X^1$, $Y^2 = \bar{Y} - Y^1$.

<u>Solving for Pareto efficiency in the Edgeworth Box</u> (Assuming differentiability and an interior solution):

Lagrangian
$$L \equiv U^1(X^1, Y^1) + \lambda[U^2(\bar{X} - X^1, \bar{Y} - Y^1) - U^{o2}]$$

$$\frac{\partial L}{\partial X^1} = \frac{\partial U^1}{\partial X^1} - \lambda\frac{\partial U^2}{\partial X^2} = 0, \text{ equivalently } \frac{\partial U^1}{\partial X^1} = \lambda\frac{\partial U^2}{\partial X^2}$$

$$\frac{\partial L}{\partial Y^1} = \frac{\partial U^1}{\partial Y^1} - \lambda\frac{\partial U^2}{\partial Y^2} = 0, \text{ equivalently } \frac{\partial U^1}{\partial Y^1} = \lambda\frac{\partial U^2}{\partial Y^2}$$

$$\frac{\partial L}{\partial \lambda} = U^2(X^2, Y^2) - U^{o2} = 0$$

This gives us then the condition

$$MRS^1_{xy} = \frac{\dfrac{\partial U^1}{\partial X^1}}{\dfrac{\partial U^1}{\partial Y^1}} = \frac{\dfrac{\partial U^2}{\partial X^2}}{\dfrac{\partial U^2}{\partial Y^2}} = MRS^2_{xy} \text{ or equivalently}$$

$$MRS^1_{xy} = \frac{\partial Y^1}{\partial X^1}\Big|_{U^1 = constant} = \frac{\partial Y^2}{\partial X^2}\Big|_{U^2 = constant} = MRS^2_{xy}$$

Pareto efficient allocation in the Edgeworth box: the slope of 2's indifference curve at an efficient allocation will equal the slope of 1's indifference curve; the points of tangency of the two curves. Exception: corner solutions, non-convex preferences (utility functions not quasi-concave).

Pareto efficient set = locus of tangencies of indifference curves
contract curve = individually rational Pareto efficient points

Market allocation in the Edgeworth Box: general equilibrium
  p_x , p_y

Household 1:Choose X^1, Y^1, to maximize $U^1(X^1,Y^1)$ subject to
$p_x X^1 + p_y Y^1 = p_x \bar{X}^1 + p_y \bar{Y}^1 = B^1$
 budget constraint is a straight line passing through the endowment point $(\bar{X}^1, \bar{Y}^1)$
with slope $-\dfrac{p_x}{p_y}$.   Similarly for Household 2.

Lagrangian for Household 1's demand determination

$L = U^1(X^1,Y^1) - \lambda [p_x X^1 + p_y Y^1 - B^1]$

$$\frac{\partial L}{\partial X} = \frac{\partial U^1}{\partial X^1} - \lambda p_x = 0$$

$$\frac{\partial L}{\partial Y} = \frac{\partial U^1}{\partial Y^1} - \lambda p_y = 0$$

Therefore, at the utility optimum subject to budget constraint we have

$$\text{MRS}^1_{xy} = \frac{\dfrac{\partial U^1}{\partial X^1}}{\dfrac{\partial U^1}{\partial Y^1}} = \frac{p_x}{p_y}; \text{ Similarly for household 2,}$$

$$\text{MRS}^2_{xy} = \frac{\dfrac{\partial U^2}{\partial X^2}}{\dfrac{\partial U^2}{\partial Y^2}} = \frac{p_x}{p_y} \quad .$$

Equilibrium prices: p^*_x and p^*_v so that
$\qquad X^{*1} + X^{*2} = \bar{X}^1 + \bar{X}^2 \equiv \bar{X}$
$\qquad Y^{*1} + Y^{*2} = \bar{Y}^1 + \bar{Y}^2 \equiv \bar{Y}$,
(market clearing)

where X^{*i} and Y^{*i}, i =1, 2, are utility maximizing mix of X and Y at prices p^*_x and p^*_y.

$$-\frac{\partial Y^1}{\partial X^1}\Big|_{U^1=U^{1*}} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{p_x}{p_y}$$

$$\frac{p_x}{p_y} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = -\frac{\partial Y^2}{\partial X^2}\Big|_{U^2=U^{2*}}$$

The price system <u>decentralizes</u> the efficient allocation decision.

The Robinson Crusoe Model

q = oyster production
c = oyster consumption
168 (hours per week) endowment
L = labor demanded
R = leisure demanded
168-R =labor supplied

$$q = F(L) \tag{2.1}$$

$$R = 168 - L \tag{2.2}$$

Centralized Allocation of the Robinson Crusoe Model
We assume second order conditions (convexity = concavity of production and utility functions= diminishing marginal product (in production) + diminishing marginal rate of substitution(in consumption)) so that local maxima are global maxima:

$$F'' < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial R^2} < 0, \frac{\partial^2 u}{\partial c \partial R} > 0. \qquad \text{(Concavity, 2}^{\text{nd}}\text{ order conditions)}$$

$$u(c,R) = u(F(L), 168 - L) \tag{2.3}$$

$$\max_{L} \; u(F(L), \; 168 - L) \tag{2.4}$$

$$\frac{d}{dL} u(F(L), \; 168 - L) = 0 \tag{2.5}$$

$$u_c F' - u_R = 0 \tag{2.6}$$

$$\left[-\frac{dq}{dR} \right]_{u=u\max} = \frac{u_R}{u_c} = F' \tag{2.7}$$

Pareto efficient

$$MRS_{R,c} = MRT_{R,q} (= RPT_{R,q})$$

General Equilibrium: Decentralized Market Allocation of the Robinson Crusoe
Model

$$\Pi = F(L) - wL = q - wL \tag{2.8}$$

Income:

$$Y = w \cdot 168 + \Pi \tag{2.9}$$

Budget constraint:

$$Y = wR + c \tag{2.10}$$

Equivalently, $c = Y - wR = \Pi + wL = \Pi + w(168-R)$, a more conventional
definition of a household budget constraint.

Firm profit maximization in the market economy:

$$\Pi = q - wL \tag{2.11}$$

$$\frac{d\Pi}{dL} = F' - w = 0, \text{ so } F'(L^0) = w \tag{2.14}$$

Household budget constraint:

$$wR + c = Y = \Pi^0 + w168 \tag{2.15}$$

Household utility optimization in the market economy:
Choose c, R to maximize u(c, R) subject to (2.15). The Lagrangian is

$$V = u(c,R) - \lambda (Y - wR - c)$$

$$\frac{\partial V}{\partial c} = \frac{\partial u}{\partial c} + \lambda = 0$$

$$\frac{\partial V}{\partial R} = \frac{\partial u}{\partial R} + \lambda w = 0$$

Dividing through, we have

$$MRS_{R,c} = \left[-\frac{dc}{dR} \right]_{u=u^*} = \frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w \tag{2.19}$$

Definition : <u>Market equilibrium of the Robinson Crusoe Model</u>. Market equilibrium consists of a wage rate w^0 so that at $w^0, q = c$ and $L = 168 - R$, where q, L are determined by firm profit maximizing decisions and c, R are determined by household utility maximization. (in a centralized solution L=168-R by definition; in a market allocation wages and prices should adjust so that as an equilibrium condition L will be equated to 168-R).

Profit maximization at w^0 implies $w^0 = F'(L^0)$. (Recall (2.14))
Utility maximization at w^o implies

$$\frac{u_R(c^0, R^0)}{u_c(c^0, R^0)} = w^0 \qquad\qquad \text{(Recall (2.19))}$$

Market-clearing implies $\qquad R^0 = 168 - L^0, c^0 = F(L^0)$.

So combining (2.14) and (2.19), we have

$$F' = \frac{u_R}{u_c} \qquad\qquad (2.25)$$

which implies Pareto efficiency. The market general equilibrium decentralizes the efficient allocation.